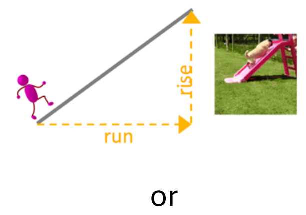
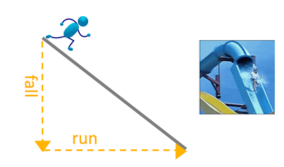
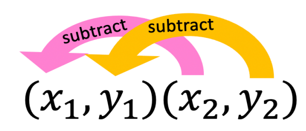
Straight Line Graphs

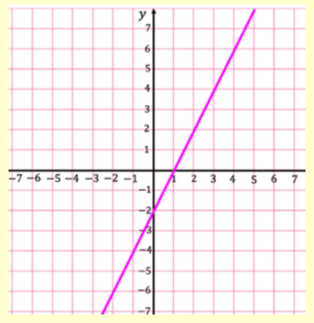
Slopes and Gradients

There are 3 ways to calculate slopes/gradients and it depends what information is given to us

Way 1: If given the graph	Way 2: If given 2 points	Way 3: If given another line
Build a triangle on the line and use the formula $slope = \frac{\text{rise or fall}}{\text{run}} = \frac{how\ much \uparrow or \downarrow}{how\ much \rightarrow}$  or  Careful: Rise comes first  If a line is going down the gradient is negative	 $slope = \frac{y_2 - y_1}{x_2 - x_1}$	A line has the form $y = mx + c$ You can read off the slope from the form  If you do not have the line in form above re-arrange first to get the following form 

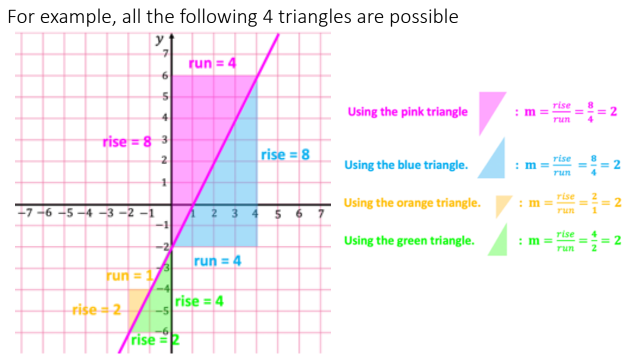
Example 1

Find the slope of the following line



It doesn't matter what size triangle we build as long as we use 'nice points' on the cross to build it

For example, all the following 4 triangles are possible

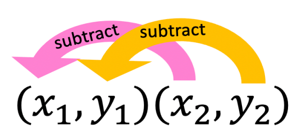


Hence, we have the slope as 2

Example 2

Find the slope of the line joining points $(-1,3)$ and $(2,4)$

Let's colour code as $(-1, 3), (2, 4)$



$$slope = \frac{4-3}{2--1} = \frac{1}{3}$$

Example 3 - Find the slope of all the following lines

$y = 2x - 1$	$y = x + 2$	$y = -x + 4$	$y = -2 + 3x$	$y = 2 - 4x$	$x = 4$	$y = 5$
$y = 2x - 1$ We can spot this straight away gradient = 2	$y = x + 2$ This means the same as $y = 1x + 2$ gradient = 1	$y = x + 2$ This means the same as $y = -1x + 4$ gradient = −1	Need to re-order this first since it is not in the form $y = mx + c$ $y = 3x - 2$ gradient = 3	Need to re-order this first $y = -4x + 2$ gradient = −4	This is a vertical line since x is the same value the whole time. This has no gradient. The gradient here is undefined	This is a horizontal line since y is the same value the whole time. $y = 5$ is like writing $y = 0x + 5$ gradient = 0

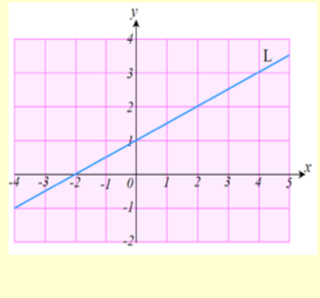
$y + x = 4$	$y - 2x = 5$	$2x + 4y = 5$	$5x - 2y = 7$	$-2x + 3y - 1 = 0$	$x + 2y + 5 = 0$	$y = \frac{x}{3} + 2$
We need to use algebra to re-arrange $y = -x + 4$ This means the same as $y = -1x + 4$ gradient = −1	We need to use algebra to re-arrange $y = 2x + 5$ gradient = 2	We need to use algebra to re-arrange $4y = -2x + 5$ $y = \frac{-2x+5}{4}$ We can split this up in order to separate the gradient and y intercept $y = -\frac{2}{4}x + \frac{5}{4}$ We can simplify the fraction $y = -\frac{1}{2}x + \frac{5}{4}$ gradient = $-\frac{1}{2}$	We need to use algebra to re-arrange $-2y = -5x + 7$ $y = \frac{-5x+7}{-2}$ We can split this up in order to separate the gradient and y intercept $y = \frac{5}{2}x - \frac{7}{2}$ gradient = $\frac{5}{2}$	We need to use algebra to re-arrange $3y = 2x + 1$ $y = \frac{2x+1}{3}$ We can split this up in order to separate the gradient and y intercept $y = \frac{2}{3}x + \frac{1}{3}$ gradient = $\frac{2}{3}$	We need to use algebra to re-arrange $2y = -x - 5$ $y = \frac{-x-5}{2}$ We can split this up in order to separate the gradient and y intercept $y = -\frac{1}{2}x - \frac{5}{2}$ gradient = $-\frac{1}{2}$	means $y = \frac{1}{3}x + 2$ gradient = $\frac{1}{3}$

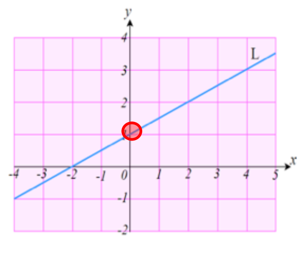
y Intercepts

Way 1: If given a graph	Way 2: Using the form $y = mx + c$
This is easy to find on a graph as it is just the point where you can see the line cross the y-axis. For the example below, it is clearly the point $(0, 1)$.	$+c$ is y-intercept. Hence, use similar methods from gradient section to rearrange equation to find c

Example 1

Find the y intercept





It is clearly the point $(0, 1)$

Example 2 - Find the y intercept of all the following lines

$y = x - 2$	$y = 2x - 1$	$y = -x + 4$	$y = -2 + 3x$	$y = 2 - 4x$	$2y - x = 2$	$3y - 2x - 4 = 0$
$y = x - 2$ y intercept is −2 $(0, -2)$	$y = 2x - 1$ y intercept is −1 $(0, -1)$	$y = -x + 4$ y intercept is 4 $(0,4)$	Need to re-order this first $y = 3x - 2$ y intercept is − 2 $(0, -2)$	Need to re-order this first $y = -4x + 2$ y intercept is 2 $(0,2)$	Need to re-order this first $2y = x + 2$ $y = \frac{1}{2}x + 1$ y intercept is 1 $(0,1)$	Need to re-order this first $3y = 2x + 4$ $y = \frac{2}{3}x + \frac{4}{3}$ y intercept is 1 $\left(0, \frac{4}{3}\right)$

Midpoints

Given two points (x_1, y_1) and (x_2, y_2)

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

In English: Add the x coordinates and divide by 2 (i.e. find the average) and add the y coordinates and divide by 2.

Distances

There are 2 ways to find the distance:

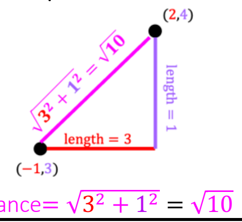
Way 1: Build a triangle- We find the x and y distances between the coordinates and use Pythagoras to find the hypotenuse length which is the distance between the points.

Way 2 Formula (which comes from how the triangle is built in Way 1)

$$\text{Given two points } (x_1, y_1) \text{ and } (x_2, y_2) \Rightarrow \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Find the distance between the 2 points $(-1,3)$ and $(2,4)$.

Let's colour code as $(-1, 3), (2, 4)$

Way 1: plot the points and build a triangle	Way 2: Formula
 Distance= $\sqrt{3^2 + 1^2} = \sqrt{10}$	Distance = $\sqrt{(2 - -1)^2 + (4-3)^2}$ $= \sqrt{3^2 + 1^2} = \sqrt{10}$

Given A Line Equation, Draw The Graph

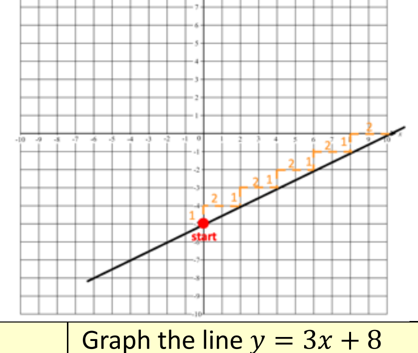
Example

Graph $y = \frac{1}{2}x - 5$

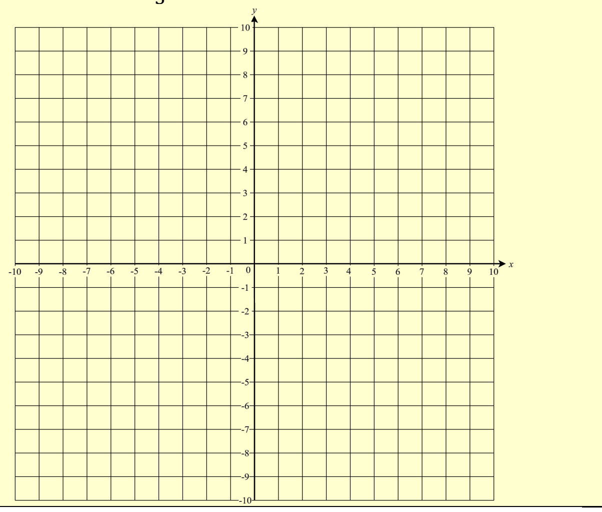
$y = \frac{1}{2}x - 5$ start

START at −5 on y axis

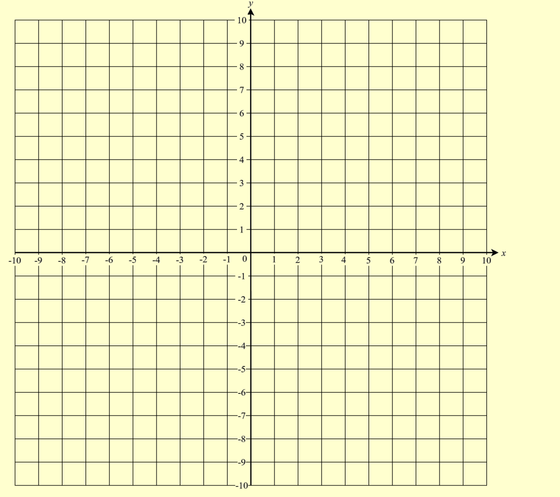
slope is positive so we rise 1 and then run 2



Graph the line $y = -\frac{4}{3}x + 8$



Graph the line $y = 3x + 8$

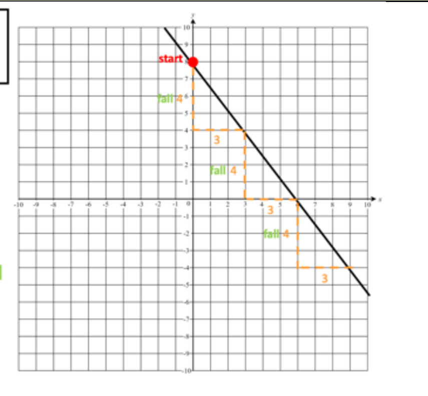


Graph $y = -\frac{4}{3}x + 8$

$y = \frac{-4}{3}x + 8$ start

START at 8 on y axis

slope is negative so we fall
Fall 4
Run 3



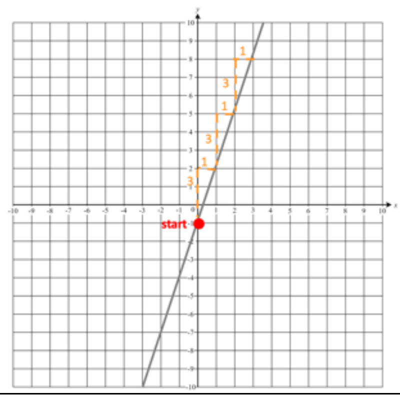
Graph $y = 3x - 1$

Since 3 is not a fraction, we have to turn it into a fraction to see rise over run

$y = \frac{3}{1}x - 1$ start

START at −1 on y axis

Slope is positive so we rise
Rise 3
Run 1



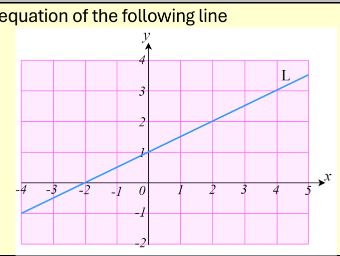
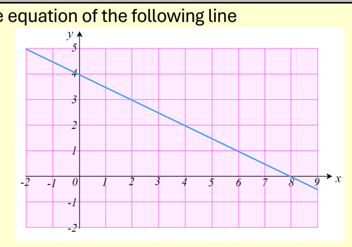
Finding The Equation Of A Line

Basics

You need two things to form the equation – the gradient m and y-intercept c

Then, the way you write the equation becomes

$$y = mx + c$$

Grade 4 example:	Grade 4 example:	Grade 5 example:
Find the equation of the following line 	Find the equation of the following line 	Find the equation of the line that passes through the points $(2, -4)$ and $(5,10)$
Slope = Pick ANY 2 points on the line and form a triangle. Let's pick the points (0,1) and (4,3) slope= $\frac{rise}{run} = \frac{2}{4} = \frac{1}{2}$ c = where the line crosses the y axis = 1 $y = \frac{1}{2}x + 1$	Slope = Pick ANY 2 points on the line and form a triangle. Let's pick the points (0,4) and (8,0) slope = $\frac{rise}{run} = \frac{-4}{8} = -\frac{1}{2}$ c = where the line crosses the y axis = 4 $y = -\frac{1}{2}x + 4$	First, we find the slope using our formula as $\frac{10 - -4}{5-2} = \frac{14}{3}$ Now, using either point, we can use point-slope form $y - 10 = \frac{14}{3}(x - 5)$

With Parallel and Perpendicular Slopes

When a line is parallel to another line, we can say the the gradient is equal this line's gradient.

When a line is perpendicular to another line, we can say the the gradient is found by flip then negate.

AND

Grade 7 Example:	Grade 7 Example	Grade 7 Example
Find the equation of the line which is parallel to $y = 3x - 8$	Find the equation of the line which is parallel to $3y + 6x = 7$and passes through the point (1,3)	What is the equation of the line that is perpendicular to the line $y = 4x - 9$, which passes through the point $(0, -3)$
Since the line is parallel, the gradient is same, which is 3. Now, the y-intercept could be anything so we write $y = 3x + c, \quad c \in \mathbb{R}$	Since the line is parallel, the gradient is same, which is $-\frac{6}{3} = -2$. Now, using point-slope form $y - 3 = -2(x - 1)$	Since the line is perpendicular, the gradient is flip and negate, which is $-\frac{1}{4}$. Now, using point-slope form $y + 3 = -\frac{1}{4}(x - 0)$

Grade 7 Example	Grade 7 Example	Grade 7 Example
Find the equation of the line which is parallel to $3y + 6x = 7$and passes through the point (1,3)	$P(-9,7)$ and $Q(11,12)$ have a **midpoint**, M. L is perpendicular to the line segment PQ. L passes through M. Find an equation for L.	We have three coordinates $A(-3,0), B(1,6)$, and $C(5,2)$. Find the equation of the line passing through C and perpendicular to AB. Give your answer in the form $ax + by = c$.
Since the lines are parallel, we know the gradients are same, so let us find the gradient of the given line first. We can use Way 3 since the equation is given here and rearrange to get $3y + 6x = 7$ $y = \frac{7}{3} - 2x$ Hence, the gradient is −2. We know it passes through the point (1,3) allowing us to form the equation $y - 3 = -2(x - 1)$	Since we know it is perpendicular, we start by finding gradient of PQ. We use Way 2 since points are given, $\frac{12-7}{11-(-9)} = \frac{1}{4}$ Hence, the gradient of the perpendicular is $m = -\frac{1}{\frac{1}{4}} = -4$ For the point, we know the line passes through the midpoint. We can find this using our formula as $\left(\frac{-9+11}{2}, \frac{7+12}{2}\right) = (1,9.5)$ So, we can form the equation $y - 9.5 = -4(x - 1)$	To find the equation, we want to start by finding the slope. This is done by using the fact that the line is perpendicular to AB. To use this, we need to find the slope of AB as Slope AB $= \frac{6-0}{1--3} = \frac{3}{2}$ Hence, we want to flip then negate giving us $m = -\frac{1}{\frac{3}{2}} = -\frac{2}{3}$ Thus, we have a slope and a point (5, 2) that the line passes through. $y - 5 = -\frac{2}{3}(x - 2)$ To get our desired form, we get rid of fractions $3y - 15 = -2x + 4$ $2x + 3y - 19 = 0$

Using Parallel and Perpendicular Gradients

Grade 7 Example	Grade 7 Example
The equation of the line L_1 is $y = 2x + 3$. The equation of the line L_2 is $5y - 10x + 4 = 0$. Show that these two lines are parallel.	A triangle has vertices $P(-3, -6), Q(1, 4)$, and $R(5, -2)$. M is the midpoint of PQ and N is the midpoint of QR. Prove MN is parallel to PR.
To show the lines are parallel, gradients must be equal. The gradient of L_1 is found by the equation $y = 2x + 3$ Hence, $m_1 = 2$. The gradient of L_2 is found by rearranging the equation $5y - 10x + 4 = 0$ $5y = 10x - 4$ $y = 2x - \frac{4}{5}$ hence $m_2 = 2.$ Since, $m_1 = 2 = m_2$, the lines are parallel.	We can find the midpoint M of PQ by finding the average of the x and y coordinates. Hence, $M = \left(\frac{-3+1}{2}, \frac{-6+4}{2}\right) = (-1, -1)$ We can find the midpoint N of RQ by finding the average of the x and y coordinates. Hence, $N = \left(\frac{5+1}{2}, \frac{-2+4}{2}\right) = (3, 1)$ Now, to show parallel lines, we need to show the slopes are equal. Slope MN $= \frac{-1-1}{-1-3} = \frac{1}{2}$ and Slope PR $= \frac{-2-6}{5-3} = \frac{1}{2}$ Hence, we can see that Slope MN $= \frac{1}{2} =$ Slope PR

Grade 7 Example

Here are the equations of four straight lines. Two of the lines are parallel. Find which two lines are parallel.

Line A: $y = 2x + 4$ *Line B*: $2y = x + 4$ *Line C*: $2x + 2y = 4$ *Line D*: $2x - y = 4$

We find the gradients of each line one at a time

The gradient of A is found by the equation	The gradient of B is found by rearranging	The gradient of C is found by rearranging	The gradient of D is found by rearranging
$y = 2x + 4$ Hence, $m_1 = 2$.	$2y = x + 4$ $y = \frac{1}{2}x + 2$ Hence, $m_2 = \frac{1}{2}$.	$2x + 2y = 4$ $y = -1x + 2$ Hence, $m_3 = -1$.	$2x - y = 4$ $y = 2x - 4$ Hence, $m_4 = 2$.

Clearly, we see $m_1 = m_4$ hence Line A and Line D are parallel.

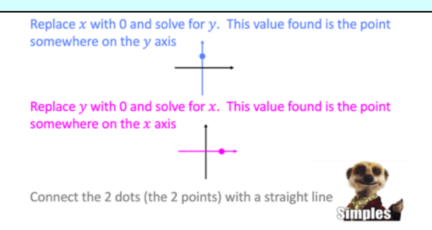
Grade 7 Example	Grade 7 Example
The equation of the line L_1 is $y = 2x - 5$. The equation of the line L_2 is $6y + kx - 12 = 0$. Given L_1 and L_2 are perpendicular, find the value of k.	The points are P(3, 4) and Q(a, b). A line perpendicular to PQ is given by $3x + 2y = 7$. Find an expression for b in terms of a
Since the lines are perpendicular, the product of their gradients must be −1. Let us find the gradients. The gradient of L_1 is found by the equation $y = 2x - 5$ Hence, $m_1 = 2$. The gradient of L_2 is found by rearranging the equation $6y + kx - 12 = 06y = -kx + 12$ $y = -\frac{k}{6}x + 2$ hence, $m_2 = -\frac{k}{6}$. So, $\left(-\frac{k}{6}\right)(2) = -1 \Rightarrow \boldsymbol{k = 3}$	Since the lines are perpendicular, we want the product of the slopes to be −1. Hence, we have the slope of PQ as Slope PQ $= \frac{b-4}{a-3}$ The slope of the perpendicular line is found by rearranging the equation as $3x + 2y = 7$ $y = -\frac{3}{2}x + \frac{7}{2}$ Hence, the slope is $-\frac{3}{2}$. So, $\left(\frac{b-4}{a-3}\right)\left(-\frac{3}{2}\right) = -1$ $3b - 12 = 2a - 6$ $\boldsymbol{b = \frac{2}{3}a + 2}$

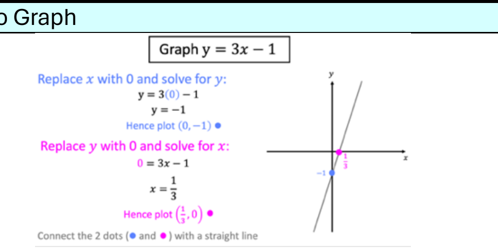
x and y Intercepts – A Quick Method

- x intercept: set $y = 0$ and solve for x
- y intercept : set $x = 0$ and solve for y

Find the y intercept of the line $2y - x = 2$	Find the x intercept of the line $2y - x = 2$
We plug in $x = 0$ $2y - 0 = 2$ $y = 1$ Hence, y-intercept is $(0, 1)$	We plug in $y = 0$ $2(0) - x = 2$ $x = -2$ Hence, the x-intercept is $(-2, 0)$

Quick Hack To Graph





The straight line **L** passes through the points $(4, -1)$ and $(6,4)$
The straight Line **M** is perpendicualr to **L** and intersects the y axis at the point (0,8)
Find the coordinates of the point where M intersects the x axis

Intersections Of Two Lines

This finds where the 2 lines cross, so we just solve simultaneously as this is what solving simultaneously finds us (using our knowledge of how to solve simultaneous equations which is not covered in this sheet).

Recall that we can use elimination or substitution to solve simultaneous equations.

Example

Find the point where the graphs of the lines $6x + 2y = -3$ and $4x - 3y = 11$ intersect.

Solving simultaneously

$$6x + 2y = -3$$
$$4x - 3y = 11$$

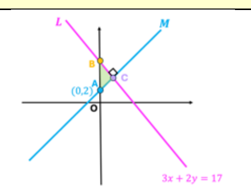
gives $x = 0.5, y = -3$ so we have the intersection point $(0.5, -3)$

Intersections and Areas

The line l_1 has equation $y = -2x + 3$. The line l_2 is perpendicular to l_1 and passes through the point (5, 6). i.Find an equation for l_2 The line l_2 crosses the x-axis at the point A and the y-axis at the point B. ii.Find the x-coordinate of A and the y-coordinate of B iii.Given that O is the origin, find the area of the triangle OAB	Find an equation of the straight line passing through the points with coordinates (4, −7) and (−6, 11),The line crosses the x-axis at point A and the y-axis at point B and O is the origin. Find the area of triangle AOB
i. We need the gradient and point to find the equation of the line Perpendicular gradient: "flip the fraction and change the sign"$\Longrightarrow \frac{1}{2}$ $y = \frac{1}{2}x + c$ Passes through B(5,6) $6 = \frac{1}{2}(5) + c$ $c = 6 - \frac{5}{2} = \frac{7}{2}$ So full equation is $y = \frac{1}{2}x + \frac{7}{2}$ ii. Crosses y axis: set $x = 0$ and solve for y $y = \frac{1}{2}(0) + \frac{7}{2}$ $y = \frac{7}{2}$ $B\left(0, \frac{7}{2}\right)$ y coordinate of B is $\frac{7}{2}$ Crosses x axis: set $y = 0$ and solve for x $y = \frac{1}{2}x + \frac{7}{2}$ $0 = \frac{1}{2}x + \frac{7}{2}$ $\frac{1}{2}x = -\frac{7}{2}$ $A(-7,0)$ hence x coordinate of A is −7 iii. So, we have the points $A(-7,0)$, $O(0,0)$, $B\left(0, \frac{7}{2}\right)$ Area $\frac{1}{2} \times base \times height = \frac{1}{2}(7)\left(\frac{7}{2}\right) = \frac{49}{4} units^2$	We need the gradient and point to find the equation of the line gradient $= \frac{11--7}{-6-4} = \frac{18}{-10} = -\frac{9}{5}$ $y = -\frac{9}{5}x + c$ Passes through (4, −7) and (−6, 11). We can sub in either point. Let's choose (4,−7) $-7 = -\frac{9}{5}(4) + c$ $-7 = -\frac{36}{5} + c$ $c = \frac{1}{5}$ So line equation is $y = -\frac{9}{5}x + \frac{1}{5}$ Crosses x axis: set $y = 0$ and solve for x $y = -\frac{9}{5}x + \frac{1}{5}$ $0 = -\frac{9}{5}x + \frac{1}{5}$ $x = \frac{1}{9}$ hence $A\left(\frac{1}{9}, 0\right)$ Crosses y axis: set $x = 0$ and solve for y $y = -\frac{9}{5}x + \frac{1}{5}$ $y = -\frac{9}{5}(0) + \frac{1}{5}$ $y = \frac{1}{5}$ hence $B\left(0, \frac{1}{5}\right)$ So, we have the points $A\left(\frac{1}{9}, 0\right)$ $O(0,0)$, $B\left(0, \frac{1}{5}\right)$ Area $\frac{1}{2} \times base \times height = \frac{1}{2}\left(\frac{1}{9}\right)\left(\frac{1}{5}\right) = \frac{1}{90} units^2$
The line $l_1$has equation $2x - 3y + 12 = 0$ cross the x axis at the point A and the y axis at the point B. The line l_2 is perpendicular to l_1 and passes through B. i. Find an equation for l_2 The line l_2 crosses the x axis at the point C ii. Find the area of triangle ABC	The line l_1 has equation $y = 3x + 2$ and the line l_2 has equation $3x + 2y - 8 = 0$. The intersection of l_1 and l_2 is P. i.Find the coordinates of P ii.The lines l_1 and l_2 cross the line $y = 1$ at the points A and B respectively iii.Find the area of triangle ABP
Need to re-arrange $2x - 3y + 12 = 0$ into the form $y = mx + c$ $3y = 2x + 12$ $y = \frac{2}{3}x + 12$ Perpendicular gradient: "flip the fraction and change the sign"$\Longrightarrow -\frac{3}{2}$ Crosses y axis: set $x = 0$ and solve for y $2(0) - 3y + 12 = 0$ $y = 4$ hence $B(0,4)$ $y = -\frac{3}{2}x + c$ Passes through B(0,4) $4 = -\frac{3}{2}(0) + c$ hence $c = 4$ So full equation is $y = -\frac{3}{2}x + 4$ To find point C: Crosses x axis so set $y = 0$ and solve for x $0 = -\frac{3}{2}x + 4$ $x = \frac{8}{3}$ hence$C\left(\frac{8}{3}, 0\right)$ To find point A: Crosses x axis so set $y = 0$ and solve for x $2x - 3(0) + 12 = 0$ $x = -6$ $A(-6,0)$ So, we have the points $A(-6,0)$, $B(0,4)$, $C\left(\frac{8}{3}, 0\right)$ Area $\frac{1}{2} \times base \times height = \frac{1}{2}\left(\frac{26}{3}\right)(4) = \frac{52}{3} units^2$	

Intersections and Areas Continued

Grade 8 Example	Grade 8 Example	Grade 8 Example
The straight line L has equation $3x + 2y = 17$. The point A has coordinates (0,2). The straight line M is perpendicular to L and p passes through A. Line L crosses the y axis at the point B. Lines L and M intersect at the point C. Work out the area of triangle ABC	L_1 and L_2 are two straight lines. The origin of the coordinate axes is O. L_1 has equation $5x + 10y = 8$. L_2 is perpendicular to L_1 and passes through the point with coordinates (8,6). L_2 crosses the x axis at the point A. L_2 intersects the straight line with equation $x = -3$ at the point B. Find the area of triangle AOB	The line l_1 passes through the points P(–1, 2) and Q(11, 8) i. Find an equation for l_1 in the form $y = mx + c$, where m and c are constants. The line l_2 passes through the point R(10, 0) and is perpendicular to l_1. The lines l_1 and l_2 intersect at the point S. ii. Calculate the coordinates of S. iii. Hence, or otherwise, find the exact area of triangle PQR



Given $3x + 2y = 17$

Need to get this into slope-intercept form

$$y = -\frac{3}{2}x + \frac{17}{2}$$

M is perpendicular hence has gradient $\frac{2}{3}$

$$y = \frac{2}{3}x + c$$

The line passes through (0,2)

$$2 = \frac{2}{3}(0) + c \text{ hence } c = 2$$

Hence the M looks like

$$y = \frac{2}{3}x + 2$$

Now we need to find where L crosses the y axis: Let $x = 0$ and solve for y

$$y = -\frac{3}{2}(0) + \frac{17}{2}$$

Hence B $(0, \frac{17}{2})$

Intersection point (solve simultaneously)

$$\frac{2}{3}x + 2 = -\frac{3}{2}x + \frac{17}{2}$$

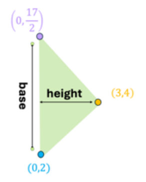
$$x = 3$$

When $x = 3$:

$$y = -\frac{3}{2}(3) + \frac{17}{2} = 4$$

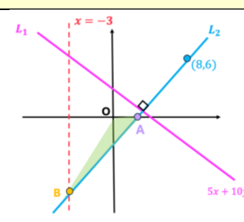
Hence C (3,4)

Now we need the area of the triangle



$$\text{Area} = \frac{1}{2} \times bh = \frac{1}{2}\left(\frac{17}{2} - 2\right)(3) = 9.75$$

Note: You could have also used the fact that that BCA is a right angle



L_1: $5x + 10y = 8$

We need to get this into slope-intercept form in order to locate the slope

$$10y = -5x + 8$$

$$y = -\frac{5}{10}x + \frac{8}{10}$$

$$y = -\frac{1}{2}x + \frac{4}{5}$$

To find the equation of L_2:

L_2 is perpendicular hence has gradient 2

$$y = 2x + c$$

The line passes through (8,6) which we can substitute in as $x = 8, y = 6$ to find c

$$6 = 2(8) + c$$

$$c = -10$$

Hence the L_2 looks like

$$y = 2x - 10$$

To find A:

Set $y = 0$

$$0 = 2x - 10$$

$$x = 5$$

Hence A(5,0)

To find B:

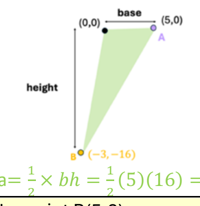
This is where $x = -3$ and $y = 2x - 10$ intersect hence we solve simultaneously

$$y = 2(-3) - 10$$

$$y = -16$$

Hence B$(-3, -16)$

We want the area of the triangle



$$\text{Area} = \frac{1}{2} \times bh = \frac{1}{2}(5)(16) = 40$$

i. We need the gradient and a point in order to find the equation of the line l_1

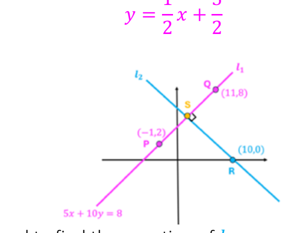
$$\text{gradient} = m = \frac{8-2}{11-1} = \frac{6}{12} = \frac{1}{2}$$

$$y = \frac{1}{2}x + c$$

The line l_1 passes through $(-1, 2)$ and $(11, 8)$. We can sub in either point to find c.

$$y = \frac{1}{2}x + \frac{5}{2}$$

ii.



We need to find the equation of l_2 now.

Perpendicular gradient hence -2

$$y = -2x + c$$

Passes through R(10, 0) so sub this point in to find c

$$0 = -2(10) + c$$

$$c = 20$$

$$y = -2x + 20$$

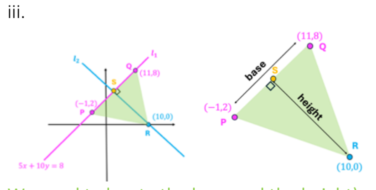
S is where the lines l_1 and l_2 intersect. To find this we need to solve $y = \frac{1}{2}x + \frac{5}{2}$ and $y = -2x + 20$ simultaneously

$$\frac{1}{2}x + \frac{5}{2} = -2x + 20$$

$$x = 7$$

$y = -2(7) + 20 = 6$ hence S(7,6)

iii.



We need to locate the base and the height)

$$\text{Area} = \frac{1}{2} \times bh = \frac{1}{2}(PQ)(RS)$$

Need the lengths RS and PQ.

$$PQ = \sqrt{(11 - -1)^2 + (8 - 2)^2} = 6\sqrt{5}$$

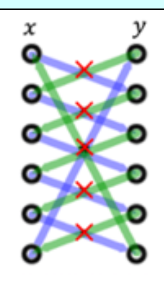
$$RS = \sqrt{(7 - 10)^2 + (6 - 0)^2} = 3\sqrt{5}$$

$$\text{Area } \frac{1}{2} \times bh = \frac{1}{2}(6\sqrt{5})(3\sqrt{5}) = 45 \text{ units}^2$$

To try: The straight line **L** passes through the point **A** $(-6,2)$ and the point B(5,3)
The straight line **M** is perpendicular to **L** and passes through the midpoing of A and B
The line M intersects the line $x = -1$ at the point C
Calculate the area of triangle ABC (answer 30.5)

Area Hack

For some questions, you use the general formulas like area of triangle and if we have a quadrilateral we split into 2 triangles. However, we have a nice **hack method** to find the area of ANY shape called the shoelace method. For this, you need all the coordinates of the shape.



Step 1: Plot the coordinates

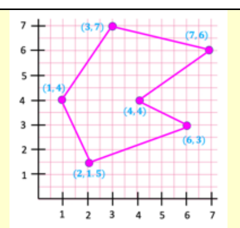
Step 2: Start at ANY coordinate

Step 3: Go **anti-clockwise** around the shape and write down all vertices as a vertical list. Make sure you "close the shape" at the end by re-writing the first coordinate you started with.

Step 4: Cross multiply corresponding diagonal coordinates and add. (First going from left to right and adding all results together and then going from right to left and adding all results together)

Step 5: Subtract these two answers and then divide by 2 hence $\frac{\searrow - \swarrow}{2}$

Find the area of the following closed shape whose vertices have the coordinates $(3,7), (7,6), (4,4), (6,3), (2,1.5), (1,4)$.



Let's pick a point to start at: $(2, 1.5)$ and write the coordinates going anti-clockwise. I colour code the points to help with the explanation. Remember, the colour coding is done by "pairing diagonal terms" like a shoelace.

(2,1.5)
(6,3)
(4,4)
(7,6)
(3,7)
(1,4)
(2,1.5)

Left bracket	Right bracket
(2,1.5)	(2,1.5)
(6,3)	(6,3)
(4,4)	(4,4)
(7,6)	(7,6)
(3,7)	(3,7)
(1,4)	(1,4)
(2,1.5)	(2,1.5)

$$\text{Area} = \frac{[(2 \times 3) + (6 \times 4) + (4 \times 6) + (7 \times 7) + (3 \times 4) + (1 \times 1.5)] - [(1.5 \times 6) + (3 \times 4) + (4 \times 7) + (6 \times 3) + (7 \times 1) + (4 \times 2)]}{2} = 17.25$$

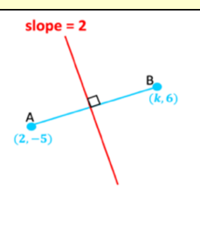
Unknown Coordinates – Summary Of Basic Formulae

Given the slope	**Given** the midpoint	**Given** the distance/length
Slope of line perpendicular to the line joining $(2, -5)$ and $(k, 6)$ is 2. Find k.	The midpoint joining $(2, -5)$ and (x, y) is $(5, -2)$. Find the value of x and y.	The distance of the line joining $(2, -6)$ and $(6, y)$ is $\sqrt{80}$. Find the value of y.

The blue line has unknown coordinate. We can form an equation based on slope, but we need to know the value of slope.

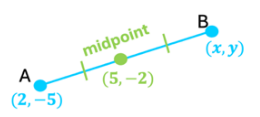
Slope of red line is 2 and blue line is perpendicular to red hence slope is $-\frac{1}{2}$

Now we can form an equation based on the slope:



$$\frac{6 - -5}{k - 2} = -\frac{1}{2}$$

$$k = -20$$



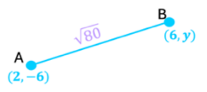
Form an equation based on the midpoint:

$$\left(\frac{2 + x}{2}, \frac{-5 + y}{2}\right) = (5, -2)$$

Hence,

$$\frac{2 + x}{2} = 5 \text{ and } \frac{-5 + y}{2} = -2$$

$$x = 8 \text{ and } y = 1$$



Form an equation based on the distance:

$$\sqrt{(6 - 2)^2 + (y - -6)^2} = \sqrt{80}$$

$$\sqrt{16 + (y + 6)^2} = \sqrt{80}$$

$$y^2 + 12y - 28 = 0$$

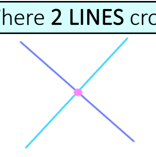
Hence,

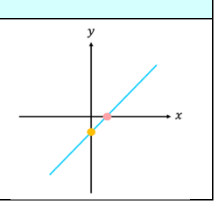
$$y = 2, y = -14$$

So, there are two possible points B

$$(6, 2), (6, -14)$$

Where **2 LINES** cross	Where **LINE** crosses AXIS
Just solve the 2 equations of the lines simultaneously!	Let $x = 0$ and solve for y *Note: This is just really solving each simultaneously. The blue line simultaneously with the line $x = 0$ hence we plug/replace $x = 0$ in the equation of the blue line* Let $y = 0$ and solve for x *Note: This is just really solving each simultaneously. The blue line with the line $y = 0$ hence we plug/replace $y = 0$ in the equation of the blue line*

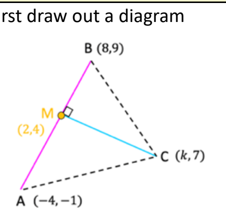




Unknown Coordinates – Using Slopes, Midpoints or Distances

The coordinates of three points are A(−4,−1) B(8,9) and C(k,7). M is the midpoint of AB and MC is perpendicular to AB. Find the value of k.	Three points have coordinates A(2,6), B(8,10) and C (6,0). The perpendicular bisector of AB meets the line BC at D. What are the coordinates of D?

First draw out a diagram



Let's find M, the midpoint of AB.

$$M = \left(\frac{-4+8}{2}, \frac{-1+9}{2}\right) = (2, 4)$$

There are two ways to move forward.

Way 1: Use perpendicular

MC is perpendicular to AB hence product of slopes is -1.

$$\text{Slope AB} = \frac{9 - -1}{8 - -4} = \frac{10}{12} = \frac{5}{6} \qquad \text{Slope MC} = \frac{4 - 7}{2 - k} = \frac{-3}{2 - k}$$

So, we say

$$\left(\frac{-3}{2 - k}\right)\left(\frac{5}{6}\right) = -1$$

$$-15 = 6(k - 2) \text{ hence } k = -\frac{1}{2}$$

Way 2: Use distances (Pythagoras)

We can use Pythagoras to build an equation and solve for k

Let's use the triangle MBC (we could have also used AMC)

$$\text{Distance MB} = \sqrt{(8 - 2)^2 + (9 - 4)^2} = \sqrt{61}$$

$$\text{Distance MC} = \sqrt{(k - 2)^2 + (7 - 4)^2} = \sqrt{(k - 2)^2 + 9}$$

$$\text{Distance BC} = \sqrt{(8 - k)^2 + (9 - 7)^2} = \sqrt{(8 - k)^2 + 4}$$

Pythagorean Theorem gives

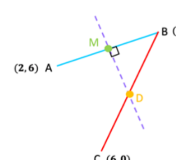
$$MB^2 + MC^2 = BC^2$$

$$61 + (k - 2)^2 + 9 = (8 - k)^2 + 4$$

Expanding and solving for k gives $k = -\frac{1}{2}$

Note: Way 1 is much easier and saves a lot of calculations. This is the benefit of working with properties of slopes.

First draw out a diagram



The midpoint between those two points is (5,8)

$$\text{slope} = \frac{10 - 6}{8 - 2} = \frac{4}{6} = \frac{2}{3}$$

Therefore, the perpendicular would have gradient by flip and negate,

$$y = -\frac{3}{2}x + c$$

c can be found by plugging in the midpoint (5,8)

$$8 = -\frac{3}{2}(5) + c$$

$$c = \frac{31}{2}$$

The perpendicular bisector equation is $y = -\frac{3}{2}x + \frac{31}{2}$

We now need the equation of the line through BC

$$\text{Gradient is } \frac{0-10}{6-8} = \frac{-10}{-2} = 5$$

$$y = 5x + c$$

c can be found by plugging in either point: B or C.

$$0 = 5(6) + c \text{ hence } c = -30$$

Hence, the equation of BC is $y = 5x - 30$.

We need to solve $y = -\frac{3}{2}x + \frac{31}{2}$ and $y = 5x - 30$ simultaneously to find D.

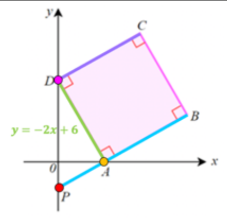
$$-\frac{3}{2}x + \frac{31}{2} = 5x - 30$$

$$x = 7$$

So $y = 5$ hence $D(7, 5)$

Unknown Coordinates – Plugging In Points Or Finding Intersections

A straight line with positive gradient passes through two points on a circle centered at the origin. One of the points has coordinates $(5, 10)$ and the other has x-coordinate -2. Find the y-intercept of this line.	A straight line with positive gradient passes through two points on a circle centered at the origin. One of the points has coordinates $(5, 10)$ and the other has x-coordinate -2. Find the y-intercept of this line.	ABCD is a trapezium with AB parallel to DC. A is the point with coordinates $(-4,6)$. B is the point with coordinates $(2,3)$. C is the point with coordinates $(-1,8)$. The trapezium has one line of symmetry. The line of symmetry intersects CD at the point E. Work out the coordinates of the point E



Line AD: $y = -2x + 6$

Need point A

Set $y = 0$ in line AD and solve for x

$$x = 3$$

$$A(3, 0)$$

Let's now find the equation of line AB

We know the line is perpendicular hence gradient $\frac{1}{2}$

$$y = \frac{1}{2}x + c$$

Passes through (3,0)

$$0 = \frac{1}{2}(3) + c$$

$$c = -\frac{3}{2}$$

$$y = \frac{1}{2}x - \frac{3}{2}$$

We can also find the point D which where the line AD crosses the y axis (y interept)

$$y = -2x + 6$$

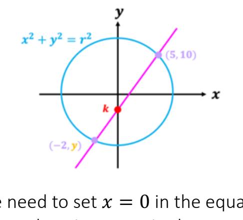
$$D(0,6)$$

Let's find point P which is where line AB meets the y axis hence we set $x = 0$:

$$y = \frac{1}{2}(0) - \frac{3}{2}$$

$$y = -\frac{3}{2}$$

Distance PD= $6 + 1.5 = 7.5$



To find k we need to set $x = 0$ in the equation of the line (or j locate the y intercept in the equation) but we don't have the equation of the line!

To find the line we need both points. To find the missing coordinate of point $(-2, y)$ we can plug it into the circle equation since it lies on the circle. But we don't have the circle equation either yet!

We know a point that lies on the circle so we can plug it in.

$$(5)^2 + (10)^2 = r^2$$

$$125 = r^2$$

Hence, the equation of the circle is

$$x^2 + y^2 = 125$$

So, we can plug $x = -2$ in now to find the y coordinate

$$(-2)^2 + y^2 = 125$$

$$y = \pm 11$$

The coordinate is below the y axis hence $y = -11$.

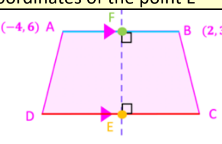
Now we can use $(-2, -11)$ and $(5,10)$ to find the line equation.

$$m = \frac{10 - -11}{5 - -2} = \frac{21}{7} = 3$$

Hence

$$y = 3x - 5$$

$$k = -5$$



Midpoint of AB

$$\left(\frac{-4 + 2}{2}, \frac{6 + 3}{2}\right) = (-1, 4.5)$$

$$\text{Gradient of AB} = \frac{3-6}{2--4} = \frac{-3}{6} = -\frac{1}{2}$$

Equation of line FE:

Perpendicular to AB hence gradient 2

$$y = 2x + c$$

Use point F to find c

$$y = 2x + 6.5$$

Equation of line CD (perpendicular) and passes through C)

$$y = -\frac{1}{2}x + 7.5$$

To find point E we solve the equations $y = 2x + 6.5$ and $y = -\frac{1}{2}x + 7.5$ simultaneously

$$x = 0.4$$

Plug back in to find y

$$y = 7.3$$

$$(0.4, 7.3)$$

Unknown Coordinates – Triangles and Quadrilaterals

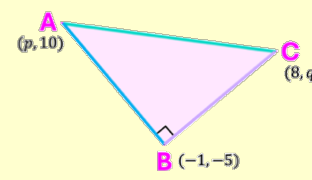
Right-Angled Triangles

- Way 1: Use fact that sides are perpendicular (slopes multiply to make -1) and form an equation based on this. Use algebra to solve for the unknown.
- Way 2: Use Pythagoras with distance formula to form an equation and then algebra to solve for unknown

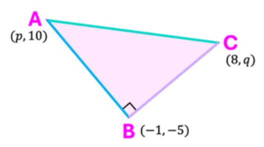
Parallelograms

- Way 1: Use fact that opposite sides are the same length by using distance formula
- Way 2: Use fact that the amount you move in x and y direction remains the same. Find it for the known adjacent coordinates then apply the same movement to find the unknown coordinate.

The three coordinates $A(p, 10), B(-1, -5)$, and $C(8, q)$ form a triangle such that angle ABC $= 90°$ where p and q are integers. Given the gradient of AC is $-\frac{6}{7}$, find p and q.



Way 1: Gradients



Slope

We know the slope of AC is $-\frac{6}{7}$ that can be found by $\frac{\text{rise}}{\text{run}}$

$$\frac{q - 10}{8 - p} = -\frac{6}{7}$$

$$7(q - 10) = -6(8 - p)$$

$$7q - 70 = -48 + 6p$$

$$7q - 6p = 22$$

Slope

$$\text{Slope AB} = \frac{10 - -5}{p - -1} = \frac{15}{p + 1} \qquad \text{Slope BC} = \frac{q - -5}{8 - -1} = \frac{q + 5}{9}$$

Since the angle is 90°, the lines are perpendicular. The product of their slopes is -1.

$$\left(\frac{15}{p + 1}\right)\left(\frac{q + 5}{9}\right) = -1$$

$$15(q + 5) = -9(p + 1)$$

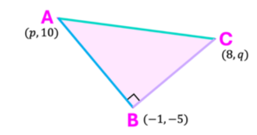
$$15q + 75 = -9p - 9$$

$$15q + 9p = -84$$

We can solve this as simultaneous equations to get

$$p = -6 \text{ and } q = -2$$

Way 2: Distance and gradient



Distance

$$AB = \sqrt{(p - -1)^2 + (10 - -5)^2} = \sqrt{p^2 + 2p + 226}$$

$$BC = \sqrt{(-1 - 8)^2 + (-5 - q)^2} = \sqrt{q^2 + 10q + 106}$$

$$AC = \sqrt{(p - 8)^2 + (10 - q)^2} = \sqrt{p^2 + q^2 - 16p - 20q + 164}$$

From the Pythagorean Theorem, we get

$$AB^2 + BC^2 = AC^2$$

$$p^2 + 2p + 226 + q^2 + 10q + 106 = p^2 + q^2 - 16p - 20q + 164$$

$$18p + 30q = -168$$

Slope

We know the slope of AC is $-\frac{6}{7}$ that is found by $\frac{\text{rise}}{\text{run}}$

$$\frac{q - 10}{8 - p} = -\frac{6}{7}$$

$$7(q - 10) = -6(8 - p)$$

$$7q - 70 = -48 + 6p$$

$$7q - 6p = 22$$

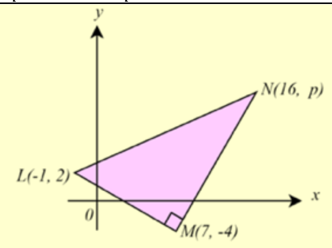
We can solve this as simultaneous equations to get

$$p = -6 \text{ and } q = -2$$

The figure shows a right-angled triangle LMN. The points L and M have coordinates (–1, 2) and (7, –4) respectively. The point N has coordinate $(16, p)$.

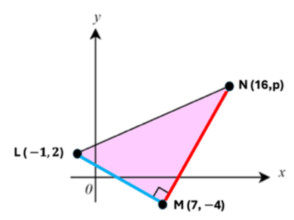
i. Find the value of p.

ii. Given that there is a point K such that the points L, M, N, and K form a **rectangle**, find the y coordinate of k



i.

Part i) Way 1: Use Perpendicular Slopes



$$\text{Slope LM} = \frac{-4 - 2}{7 - 1} = -\frac{6}{8} = -\frac{3}{4}$$

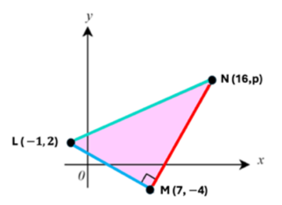
$$\text{Slope NM} = \frac{p - -4}{16 - 7} = \frac{p + 4}{9}$$

Perpendicular means the product of gradients is -1. So,

$$\left(-\frac{3}{4}\right)\left(\frac{p + 4}{9}\right) = -1$$

$$p = 8$$

Way 2: Use Distances with Pythagoras



$$LM = \sqrt{(7 - -1)^2 + (-4 - 2)^2} = \sqrt{100} = 10$$

$$MN = \sqrt{(16 - 7)^2 + (p - -4)^2} = \sqrt{81 + (p + 4)^2}$$

$$LN = \sqrt{(16 - -1)^2 + (p - 2)^2} = \sqrt{289 + (p - 2)^2}$$

$$LM^2 + MN^2 = LN^2$$

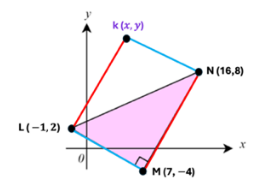
$$10^2 + 81 + (p + 4)^2 = 289 + (p - 2)^2$$

$$100 + 81 + p^2 + 8p + 16 = 289 + p^2 - 4p + 4$$

$$p = 8$$

ii.

Way 1: Parallel slopes



$$\text{Slope KN} = \frac{8 - y}{16 - x}, \text{Slope LM} = \frac{-4 - 2}{7 - -1} = -\frac{3}{4}$$

Hence, parallel means

$$\frac{8 - y}{16 - x} = -\frac{3}{4} \Rightarrow 3x + 4y = 80$$

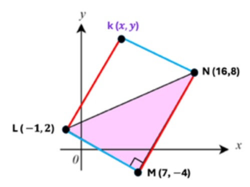
$$\text{Slope KL} = \frac{y - 2}{x - 1} = \frac{y - 2}{x + 1}, \text{Slope MN} = \frac{8 - -4}{16 - 7} = \frac{4}{3}$$

Hence, parallel means

$$\frac{y - 2}{x + 1} = \frac{4}{3} \Rightarrow 4x - 3y = -10$$

Solve simultaneously to get $x = 8, y = 14$

Way 2: Perpendicular slopes



$$\text{Slope KN} = \frac{8 - y}{16 - x}, \text{Slope MN} = \frac{8 - -4}{16 - 7} = \frac{4}{3}$$

Perpendicular means

$$\left(\frac{8 - y}{16 - x}\right)\left(\frac{4}{3}\right) = -1 \Rightarrow 3x + 4y - 80 = 0$$

$$\text{Slope KL} = \frac{y - 2}{x - -1} = \frac{y - 2}{x + 1}, \text{Slope LM} = \frac{-4 - 2}{7 - -1} = -\frac{3}{4}$$

Perpendicular means

$$\left(\frac{y - 2}{x + 1}\right)\left(-\frac{3}{4}\right) = -1 \Rightarrow 4x - 3y + 10 = 0$$

Solve simultaneously to get $x = 8, y = 14$

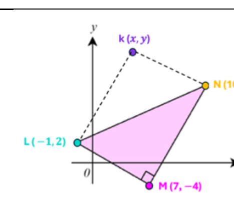
Way 3: Symmetry

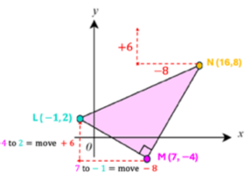
$$16 - x = 7 - -1, \qquad 8 - y = -4 - 2$$

Solving each gives

$$x = 8, y = 14$$

Note: This is true since how we move from one point to another must be the same on both sides





Way 4: Distances

We could use the fact that

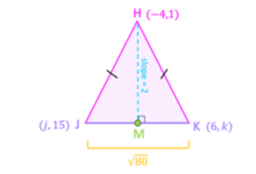
LK = MN and LM = KN

BUT…

It is not a good idea to use distance alone when there is an unknown in both coordinates. The equations are very messy to solve simultaneously. This is because there are square roots and quadratics involved

Unknown Coordinates – Distances and Shape Properties

Triangle HJK is isosceles with HJ=HK and JK= $\sqrt{80}$. H is the point with coordinates $(-4,1)$. J is the point with coordinates $(j, 15)$ where $j < 0$. K is the point with coordinates $(6, k)$. M is the midpoint of JK. The gradient of HM is 2. Find the value of j and the value of k	ABCD is a kite, with diagonals AC and BD drawn on a centimetre grid, with a scale of 1 cm for 1 unit on each axis. A is the point with coordinates $(-3, 4)$. The diagonals of the kite intersect at the point M with coordinates $(0,2)$. Given that AB = AD = 6.5 cm and the x coordinate of B is positive, find the coordinates of the points B and D



We need to build equations based on the slope and the distance

Let's first find the midpoint M of JK:

$$M = \left(\frac{j+6}{2}, \frac{15+k}{2}\right)$$

The gradient of HM is 2

Find gradient JK and use the fact it is-½

$$\frac{k - 15}{6 - j} = -\frac{1}{2}$$

$$j = 2k - 24$$

We also know the distance of JK is $\sqrt{80}$

$$\sqrt{(6 - j)^2 + (k - 15)^2} = \sqrt{80}$$

$$j^2 - 12j + k^2 - 30k + 181 = 0$$

We also know that $j = 2k - 24$

$$(2k - 24)^2 - 12(2k - 24) + k^2 - 30k + 181 = 0$$

$$k = 11, k = 19$$

When

$k = 11: j = 2(11) - 24 = -2$

$k = 19: j = 2(10) - 24 = 14$

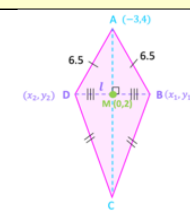
$j < 0$ (given) so $k = 11, j = -2$

Find gradient of AM to know BD

$$\frac{4 - 2}{-3 - 0} = \frac{2}{-3} = -\frac{2}{3}$$

Hence gradient of BD = $\frac{3}{2}$.

$$y = \frac{3}{2}x + 2$$



We know B is the intersection of line DB and AB so we could solve the equations for these lines simultaneously to find B, but this doesn't help since we can't find the equation of line AB as we don't know the gradient of AB

Instead we can use length of AB is 6.5

$$\sqrt{(x - -3)^2 + (y - 4)^2} = 6.5$$

$$(x + 3)^2 + (y - 4)^2 = 6.5^2$$

We need another equation to solve simultaneously with since we have 2 unknowns. Let's use equation BD

$$(x - -3)^2 + \left(\frac{3}{2}x + 2 - 4\right)^2 = 6.5^2$$

Expand and solve

$$x = 3, x = -3$$

Plug back in to find y

$$(3, 6.5), (-3, -2.5)$$

Unknown Coordinates – Ratios ("counting questions")

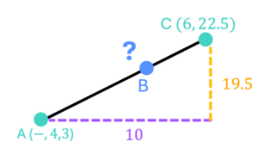
This is just an extension of what we did in Way 2 of Parallelograms where we see the "movement" in the x and y direction. Now, we introduce the concept of ratios. This is best explained using some examples.

ABC is a straight line with $AB: AC = 4: 5$. Given $A(-4, 3)$ and $C(6, 22.5)$, find the coordinates of B.

Let's draw the points out and work out the distances we know.

These are the horizontal and vertical distances between the points.

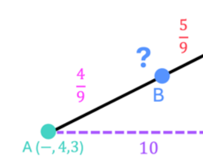
Now let's use the ratio given to us. We could go about it formulaically, or through direct logic.



Way 1: Using common sense

$$\frac{4}{9}(10) = \frac{40}{9}$$

$$\frac{4}{9}(9.5) = \frac{26}{3}$$



$$B = \left(-4 + \frac{40}{9}, 3 + \frac{26}{3}\right) = \left(\frac{4}{9}, \frac{35}{3}\right)$$

Way 2: Formulaic way

Final = Starting + Ratio(Difference in coordinates)

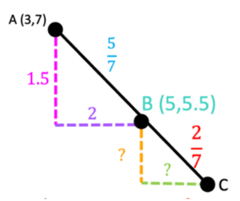
x	y
$x = -4 + \frac{4}{9}(6 - -4)$ $= \frac{4}{9}$	$y = 3 + \frac{4}{9}(22.5 - 3)$ $= \frac{35}{3}$

Hence, $B = \left(\frac{4}{9}, \frac{35}{3}\right)$

ABC is a straight line with $AB: AC = 5: 2$. Given A$(3, 7)$ and B$(5, 5.5)$, find the coordinates of C.

This time, we want the entire distance and know a part of it. Here, the formulaic way starts being more helpful.

Way 1: Using common sense



$$\frac{5}{7} \text{ represents } 2 \text{ in } x \Rightarrow \frac{1}{7} \text{ represents } \frac{2}{5} \text{ in } x \Rightarrow \frac{2}{7} \text{ represents } \frac{4}{5} \text{ in } x$$

$$\frac{5}{7} \text{ represents } 1.5 \text{ in } y \Rightarrow \frac{1}{7} \text{ represents } \frac{1.5}{5} \text{ in } y \Rightarrow \frac{2}{7} \text{ represents } \frac{3}{5} \text{ in } y$$

Hence, $C = (5 + 0.8, 5.5 - 0.6) = (5.8, 4.9)$

Way 2: Formulaic way

Final = Starting + Ratio(Difference)

x	y
$5 = 3 + \frac{5}{7}(x - 5)$ $x = 5.8$	$5.5 = 7 + \frac{5}{7}(y - 7)$ $y = 4.9$

Hence, $C = (5.8, 4.9)$

Unknown Coordinates - Summary

1) **Draw out a diagram.** Any points that are unknown and not labelled call them (x, y)
2) Are **gradients** involved? – can I form an equation based on the unknown coordinates and set it equal to the gradient (**parallel** means equal and **perpendicular** means negative reciprocal)

$$(x_1, y_1), (x_2, y_2) \Rightarrow \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

3) Are **midpoints** involved? – can I form an equation based on the unknown coordinates

$$(x_1, y_1), (x_2, y_2) \Rightarrow \text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Add x coordinates and divide by 2 (find the average) and add y coordinates and divide by 2 (find the average)

4) Are **distances** involved? – can I form an equation based on the unknown coordinates

$$(x_1, y_1), (x_2, y_2) \Rightarrow \text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Note: You can use Pythagoras instead of this formula if you wish

Can I find the **equation of a line –** don't find the equation if there are too many unknowns to find c (unless absolutely nothing else to do)
Do I want the points where **2 lines meet? Simultaneous equations**
Can I **plug** in a **point** (if given the full equation to plug into)
Can I **use symmetry** (by 'counting' horizontal & vertical lengths)

Remember: if there are 2 unknowns in the question you need 2 equations!!!

Hardest Examples With Distances

Given that

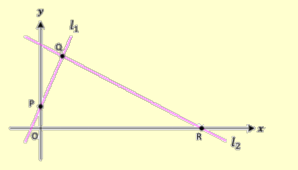
- point A has coordinates (4, 2)
- B has coordinates (15, 7)
- A line l_1 passes through A and B

i. find an equation for l_1, giving your answer in the form $px + qy + r = 0$ where p, q and r are integers to be found.

The line l_2 passes through A and is parallel to the x-axis. The point C lies on l_2 so that the length of BC is $5\sqrt{5}$

ii. Find both possible pairs of coordinates of the point

The points P(0, 2) and Q (3, 7) lie on the line l_1



The line l_2 is perpendicular to l_1, passes through Q and crosses the x-axis at the point R, as shown.

i. Find an equation for l_2, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers,

ii. the exact coordinates of R

iii. the exact area of the quadrilateral ORQP, where O is the origin

The line l_1 has equation $4y + 3 = 2x$.

The point A $(p, 4)$ lies on l_1

i. Find the value of the constant p

The line l_2 passes through the point C (2, 4) and is perpendicular to l_1

ii. Find an equation for l_2 giving your answer in the form ax + by + c = 0, where a, b c are integers

The line l_1 and the line l_2 intersect at the point D

iii. Find the coordinates of the point D

iv. Show that the length of CD is $\frac{3}{2}\sqrt{5}$

A point B lies on l_1 and the length of AB = $\sqrt{80}$

The point E lies on l_2 such that the length of the line CDE = 3 times the length of CD

v. Find the area of the quadrilateral ACBE